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Sizing Vertical Ground Heat Exchangers

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Sizing vertical ground heat exchangers (VGHEs) remains a difficult task and cannot be done with a simple rule of thumb, especially for fields with multiple boreholes. A decade ago, Philippe et al.,[1] presented in *ASHRAE Journal* a spreadsheet tool to size geothermal bore fields based on the models available at that time. This article introduces significant enhancements to this new tool based on recent developments in the field.

The new spreadsheet tool, called GHXSizing, proposes a multilevel approach in which different methods can be used depending on the amount of information available to the designer. This article provides details on the development of GHXSizing's governing sizing equations to complete the information given in the *2019 ASHRAE Handbook—Applications* and presents a number of examples of the use of GHXSizing.

Five Levels of Sizing Models

A multitude of VGHE sizing models are available. Ahmadfard and Bernier[2] characterized these models into five levels of various complexities, L0 to L4. L0 and L1 are based on rules of thumb and two peak pulses, respectively, and will not be discussed here because of their inherent inaccuracies. As illustrated schematically in *Figure 1*, this article presents the use of GHXSizing with five sizing methods in the L2 to L4 categories. As shown in the right column of *Figure 1*, GHXSizing can also evaluate g-functions, or ground thermal response factors, which are often used in the methods presented here and which can also be used to assess the temperature penalty required by some sizing models. An upcoming article will address the use and generation of g-functions more specifically.

Because they use only three ground load pulses (a minimum of information), L2 sizing models are considered low-level methods. In these models, peak building loads are used to calculate peak ground loads using the estimated heat pump coefficient of performance (COP); monthly ground loads are obtained using an estimate of the operating time during the peak month; and average yearly ground load can be determined based on equivalent full load hours (EFLH).[3] Three L2 methods will be presented: the classic ASHRAE sizing equation presented in the *2019 ASHRAE Handbook—HVAC Applications*;[4] the modified ASHRAE sizing model introduced by Philippe et al.;[1] and an alternative method, based solely on g-functions, first introduced in the *2019 ASHRAE Handbook—HVAC Applications*.[5] The classic and

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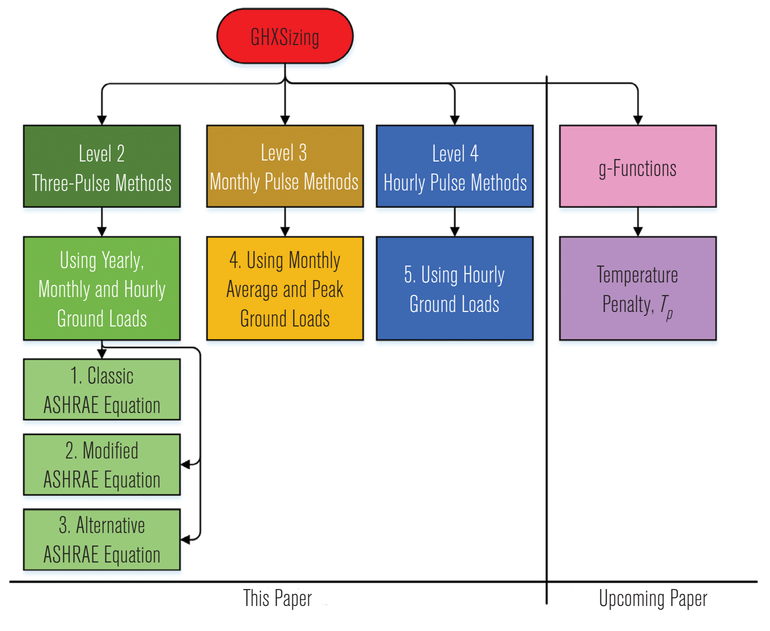
modified ASHRAE sizing equations both use a temperature penalty to account for borehole thermal interaction, but they evaluate it with different methods.

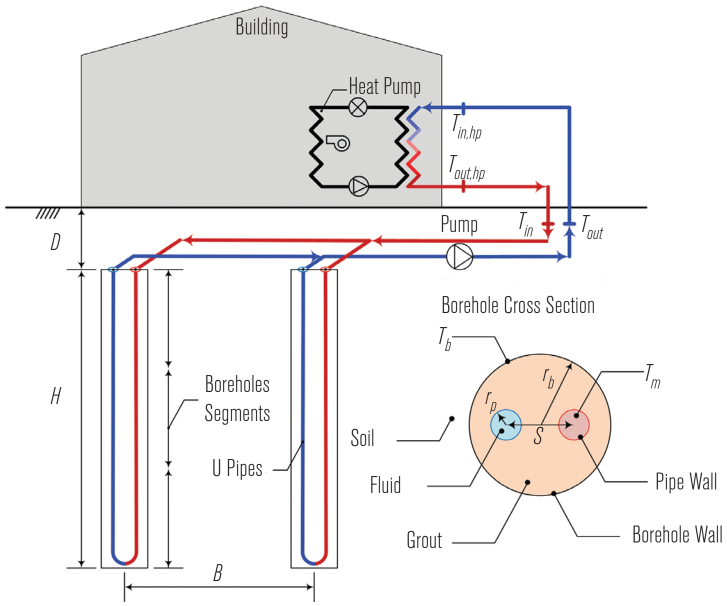
FIGURE 1 Flowchart of sizing and simulation tools provided by GHXSizing.



The monthly and hourly methods (L3 and L4) are higher-level sizing methods. The first is based on 36 ground loads (12 monthly average loads and two sets of 12 peak loads, one for heating and one for cooling), while the second is based on hourly ground loads.

Depending on the information available, L2 to L4 methods can be used with GHXSizing. Three-pulse methods are more appropriate at the early design phase when limited building load information is available, while hourly methods could be used in the final design stage. As might be expected, the required effort and calculation time are greater for higher-level methods. However, as will be shown, simple low-level methods can give accurate results when used correctly.

FIGURE 2 A typical ground-source heat pump system.



Basic Theory on Borehole Sizing

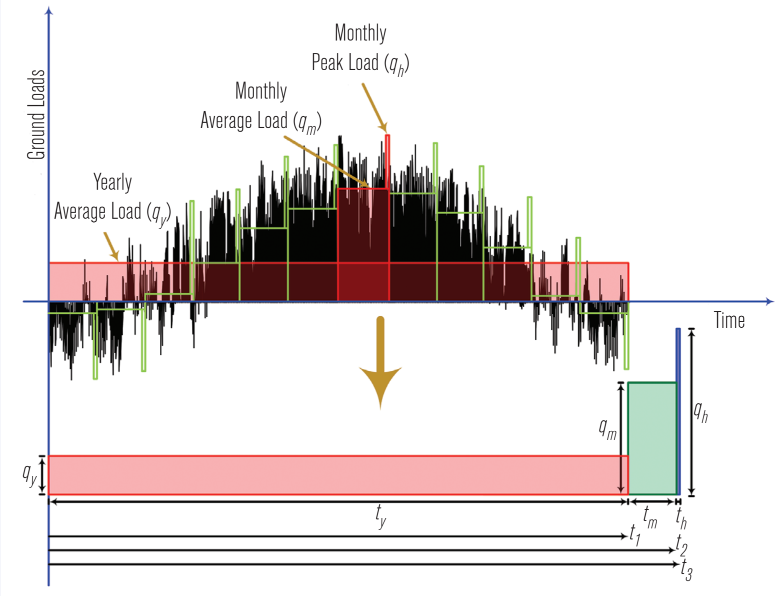
The important design parameters are shown in *Figure 2*. In this figure, D is the borehole buried depth, H is the borehole length, B is the borehole spacing, r_b is the borehole radius, r_p is the radius of the U-tube pipes and S is the center-to-center pipe distance in the borehole. The ground heat exchangers are usually linked in parallel, the total flow rate is assumed to be divided equally among all boreholes, and each borehole has the same inlet temperature, T_{in}. Furthermore, the heat pump inlet temperature, $T_{in,hp}$, is assumed to be equal to the average of the outlet temperatures of all boreholes, T_{out}. T_b is borehole wall temperature, and T_m is mean fluid temperature in the borehole.

Borehole sizing involves both finding a borehole length that will ensure that $T_{in,hp}$ remains within certain limits to avoid heat pump operational problems and limiting installation costs.

The ground loads shown in *Figure 3* are used here to demonstrate the use of L2 to L4 methods based on the loads. The top portion of the figure shows hourly ground loads. Superimposed on these ground loads are 24 horizontal lines representing 12 monthly average loads and 12 monthly peak hourly loads calculated from the hourly loads. The three ground load pulses based on these hourly values are shown on the bottom portion of the figure, where the annual peak load, q_h, the monthly average load, q_m, and the yearly average load, q_y (i.e., the annual average heat transfer to ground), are shown.

They are assumed to prevail from $t_0 = 0$ to t_1, from t_1 to t_2 and from t_2 to t_3, respectively. Typically, t_1, t_2 and t_3 correspond to 10 years, 10 years + 1 month, and 10 years + 1 month + 6 hours, respectively. In practice, the three ground loads determined in the early design phase may differ from those defined with the final hourly values.

FIGURE 3 Hourly load profile decomposed into monthly loads and three load pulses.



The following paragraphs will give some theoretical background on the origin of L2 methods. The bore field sizing methods presented here use the principle of temporal superposition of loads.[6] With reference to *Figure 3*, this principle can be illustrated by first examining the borehole wall temperature, T_b, at times t_1 and t_3 and resulting from heat injection, q_y/L, of 10 W/m (10.4 Btu/h·ft) for the first 3,650 days (0 to t_1) followed by no heat injection ($q_y/L = 0$) for the next 30.25 days from t_1 to t_3. Using typical borehole and ground conditions:

at t_1 $$T_b - T_g = \frac{q_y \Gamma(t_1 - t_0)}{L} = 10 \times 0.3809 = 3.81°\text{C} \qquad (1a)$$

at t_3 $$T_b - T_g = \frac{q_y \Gamma(t_3 - t_0) - q_y \Gamma(t_3 - t_1)}{L} \qquad (1b)$$
$$= 10 \times (0.3812 - 0.2286) = 1.53°\text{C}$$

where T_g is the ground temperature, Γ, is a ground thermal response (the so-called infinite cylindrical source analytical solution is used is used here but other methods could be used as well) evaluated over a certain time period and L is the overall bore field length (H) times the number of boreholes). Thus, T_b is 3.81°C and 1.53°C (7.0°F and 2.74°F) higher than T_g after 3,650 days and 3,680.25 days, respectively. This shows that the ground “recovers” when heat injection is stopped from t_1 to t_3. For *Equation 1b*, the influence of q_y is first calculated for the full duration ($t_3 - t_0$) and then its influence over the period ($t_3 - t_1$) is subtracted according to the principle of superposition.

Equation 1b determines T_b at time t_3 for $q_y = 0$ from t_1 to t_3, ignoring the effects of the other two ground pulses (q_m and q_h) on the final borehole temperature at t_3. In practice, these ground loads are nonzero from t_1 to t_3. Introducing the monthly average (from t_1 to t_2) and peak hourly (from t_2 to t_3) ground loads, the borehole wall temperature at time t_3 is then given by:

$$T_b - T_g = \frac{q_y\left(\Gamma(t_3 - t_0) - \Gamma(t_3 - t_1)\right) + q_m\left(\Gamma(t_3 - t_1) - \Gamma(t_3 - t_2)\right) + q_h\left(\Gamma(t_3 - t_2)\right)}{L} \qquad (2)$$

Equation 2 can be reformulated to give the bore field length as a function of the mean fluid temperature:

$$L = \frac{q_y\left(\Gamma(t_3 - t_0) - \Gamma(t_3 - t_1)\right) + q_m\left(\Gamma(t_3 - t_1) - \Gamma(t_3 - t_2)\right) + q_h\left(\Gamma(t_3 - t_2)\right) + q_h R_b}{T_m - T_g} \qquad (3)$$

where R_b is the borehole thermal resistance (between T_b and T_m) and T_m is the mean fluid temperature in the borehole. When hourly loads are used, it is more convenient to transform *Equation 3* into the following form:

$$L = \frac{\sum_{i=1}^{N}\left(q_i - q_{(i-1)}\right)\Gamma\left(t_n - t_{(i-1)}\right) + q_h R_b}{T_m - T_g} \qquad (4)$$

where q_i and q_{i-1} are the i^{th} and $(i-1)^{\text{th}}$ hourly ground loads, t_n is the total time duration of all loads from t_0 to the last load, q_N, and t_{i-1} is the time duration of loads from t_0 to the end of q_{i-1}.

Classic ASHRAE Sizing Equation

The classic ASHRAE sizing equation (*Equation 5*) first appeared in the *1995 ASHRAE Handbook—HVAC Applications*.[7,8] It is based on the approach described in *Equation 3*, and it can be used for either heating or cooling applications (with proper signs for the loads). $T_{in,hp}$ is the heat pump inlet design temperature limit in heating or cooling, and $T_{out,hp}$ is determined from an energy balance on the bore field at peak conditions.

$$L = \frac{q_y\left[\left(G_{(t_3)} - G_{(t_3-t_1)}\right)/k_g\right] + \left(q_h - W\right)\left(R_b + PLF_m\left[\left(G_{(t_3-t_1)} - G_{(t_3-t_2)}\right)/k_g\right] + F_{sc}\left[\left(G_{(t_3-t_2)}\right)/k_g\right]\right)}{T_g - \frac{\left(T_{in,hp} + T_{out,hp}\right)}{2} - T_p} \qquad (5)$$

In this equation, the annual, monthly and peak load pulses are given by (1) q_y, the net annual average heat transfer to the ground; (2) $(q_h - W)PLF_m$, the monthly average heat transfer to the ground; and (3) $(q_h - W)$, the peak hourly heat transfer rate to the ground. Note that q_y is a ground load evaluated using the building block load, heating and cooling EFLHs and heat pump COPs. The value of q_h is a peak building load converted into a ground load by subtracting the compressor power, W, which is obtained using the heat pump COP. PLF_m is the part-load factor during the design month, k_g is the ground thermal conductivity and finally F_{sc} is the short circuit heat loss factor between the upward and downward pipes in the borehole.

This last value, which is typically very close to 1, is tabulated in the *2019 ASHRAE Handbook—HVAC Applications*.[9] The three terms in brackets correspond to R_y, R_m and R_h (i.e., the yearly, monthly and hourly effective ground thermal resistances). They are evaluated using the infinite cylindrical source (ICS) analytical solution, which is often referred to as a G-factor. The temperature penalty, T_p, can be regarded as the increase/decrease of the borehole wall temperature caused by the annual ground thermal imbalance, q_y, and borehole interaction. Values of T_p are tabulated in the *2015 ASHRAE Handbook—HVAC Applications*[10] for a limited number of bore field configurations and annual ground thermal imbalances, q_y. The *2019 ASHRAE Handbook—HVAC Applications*[11] suggests an iterative method for the determination of T_p. This method is implemented in GHXSizing to determine T_p for rectangular bore fields.

Example 1 used in the *2019 ASHRAE Handbook—HVAC Applications*[12] will now be solved with GHXSizing to illustrate the use of the classic ASHRAE sizing equation. Input values and results are shown in *Table 1*. The input parameters can be entered in GHXSizing in both SI and I-P units, and results are calculated based on the selected unit system. In this article, examples are solved with SI units.

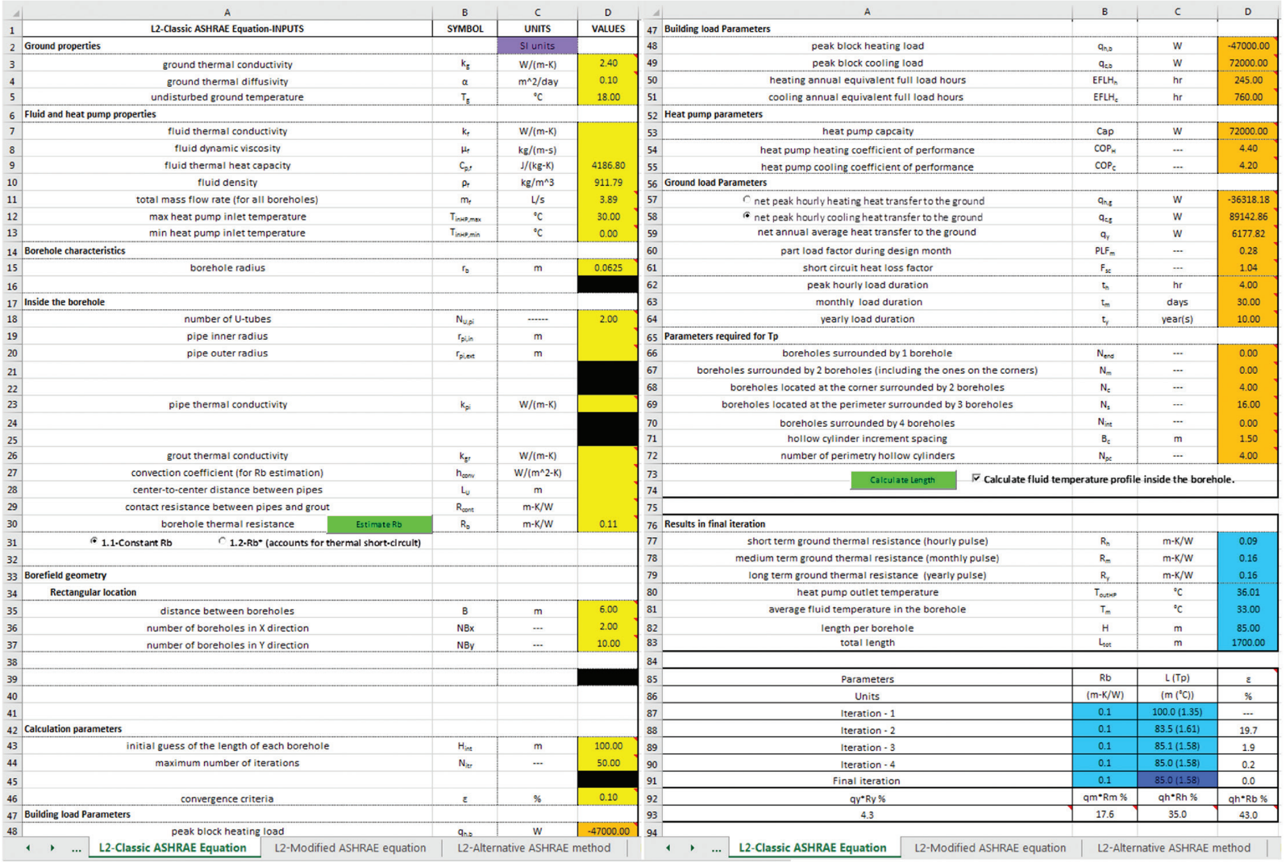
As explained in the GHXSizing instructions tab, yellow cells are input values typically used in all sizing methods. For example, cells D36 and D37 in *Table 1* indicate that a 10 × 2 bore field has been selected. Orange cells are values used for a particular method, and blue cells present results. Green buttons are used to calculate the borehole thermal resistance and the borehole length. As shown in cells D83 and C91 in *Table 1*, the borehole length obtained after five iterations is 85.0 m (279 ft) (for a total of 1700 m [5,580 ft] for the entire bore field), and T_p = 1.58°C (2.8°F). These results are the same as those of Example 1 in the *2019 ASHRAE Handbook—HVAC Applications*.[12] In some cases, there may be slight differences because GHXSizing calculates the effective ground thermal resistances using the Cooper relationship,[13] while the *Handbook* uses interpolated values on a G-factor curve.

In this example, the borehole thermal resistance, R_b, is set to 0.11 m·K/W (0.19 h·ft·°F/Btu) in cell D30. However, it is possible to let GHXSizing calculate using the so-called first order multipole method.[14] This can be done by filling cells D18 to D29 and clicking the "Estimate R_b" button in cell A30. If thermal short circuit between the downward and upward pipes is suspected (for long boreholes and/or low flow rates), then it is suggested to calculate the effective borehole thermal resistance R_b*[14] by selecting the second option in cell A31. For these cases, it may also be interesting to calculate the fluid temperature profile inside the borehole by checking the box near the "Calculate Length" button in cell A73. It is worth noting that the method to calculate T_p is relatively sensitive to the number of hollow cylinders, N_{pc}, and their increment spacing, B_c.

Modified ASHRAE Sizing Equation

Given the uncertainty in the determination of T_p and F_{sc} with the classic ASHRAE equation, a modified version of this equation has been proposed[1] (*Equation 6*). As in the classic ASHRAE equation (*Equation 5*), the three terms

TABLE 1 Input parameters and results for the example provided in the *2019 ASHRAE Handbook—HVAC Applications*[12] obtained using the classic ASHRAE equation.



	A	B	C	D
1	**L2-Classic ASHRAE Equation-INPUTS**	**SYMBOL**	**UNITS**	**VALUES**
2	**Ground properties**		SI units	
3	ground thermal conductivity	k_g	W/(m-K)	2.40
4	ground thermal diffusivity	α	m^2/day	0.10
5	undisturbed ground temperature	T_g	°C	18.00
6	**Fluid and heat pump properties**			
7	fluid thermal conductivity	k_f	W/(m-K)	
8	fluid dynamic viscosity	μ_f	kg/(m-s)	
9	fluid thermal heat capacity	$C_{p,f}$	J/(kg-K)	4186.80
10	fluid density	ρ_f	kg/m^3	911.79
11	total mass flow rate (for all boreholes)	m_f	L/s	3.89
12	max heat pump inlet temperature	$T_{inHP,max}$	°C	30.00
13	min heat pump inlet temperature	$T_{inHP,min}$	°C	0.00
14	**Borehole characteristics**			
15	borehole radius	r_b	m	0.0625
16				
17	**Inside the borehole**			
18	number of U-tubes	$N_{u,pi}$	------	2.00
19	pipe inner radius	$r_{pi,in}$	m	
20	pipe outer radius	$r_{pi,ext}$	m	
21				
22				
23	pipe thermal conductivity	k_{pi}	W/(m-K)	
24				
25				
26	grout thermal conductivity	k_{gr}	W/(m-K)	
27	convection coefficient (for Rb estimation)	h_{conv}	W/(m^2-K)	
28	center-to-center distance between pipes	L_U	m	
29	contact resistance between pipes and grout	R_{cont}	m-K/W	
30	borehole thermal resistance Estimate Rb	R_b	m-K/W	0.11
31	● **1.1-Constant Rb** ○ **1.2-Rb* (accounts for thermal short-circuit)**			
32				
33	**Borefield geometry**			
34	**Rectangular location**			
35	distance between boreholes	B	m	6.00
36	number of boreholes in X direction	NBx	---	2.00
37	number of boreholes in Y direction	NBy	---	10.00
38				
39				
40				
41				
42	**Calculation parameters**			
43	initial guess of the length of each borehole	H_{int}	m	100.00
44	maximum number of iterations	N_{itr}	---	50.00
45				
46	convergence criteria	ε	%	0.10
47	**Building load Parameters**			
48	peak block heating load	$q_{h,b}$	W	-47000.00

◂ ▸ ... **L2-Classic ASHRAE Equation** | L2-Modified ASHRAE equation | L2-Alternative ASHRAE method

	A	B	C	D
47	**Building load Parameters**			
48	peak block heating load	$q_{h,b}$	W	-47000.00
49	peak block cooling load	$q_{c,b}$	W	72000.00
50	heating annual equivalent full load hours	$EFLH_h$	hr	245.00
51	cooling annual equivalent full load hours	$EFLH_c$	hr	760.00
52	**Heat pump parameters**			
53	heat pump capcaity	Cap	W	72000.00
54	heat pump heating coefficient of performance	COP_H	---	4.40
55	heat pump cooling coefficient of performance	COP_C	---	4.20
56	**Ground load Parameters**			
57	○ net peak hourly heating heat transfer to the ground	$q_{h,g}$	W	-36318.18
58	● net peak hourly cooling heat transfer to the ground	$q_{c,g}$	W	89142.86
59	net annual average heat transfer to the ground	q_y	W	6177.82
60	part load factor during design month	PLF_m	---	0.28
61	short circuit heat loss factor	F_{sc}	---	1.04
62	peak hourly load duration	t_h	hr	4.00
63	monthly load duration	t_m	days	30.00
64	yearly load duration	t_y	year(s)	10.00
65	**Parameters required for Tp**			
66	boreholes surrounded by 1 borehole	N_{end}	---	0.00
67	boreholes surrounded by 2 boreholes (including the ones on the corners)	N_m	---	0.00
68	boreholes located at the corner surrounded by 2 boreholes	N_c	---	4.00
69	boreholes located at the perimeter surrounded by 3 boreholes	N_s	---	16.00
70	boreholes surrounded by 4 boreholes	N_{int}	---	0.00
71	hollow cylinder increment spacing	B_c	m	1.50
72	number of perimetry hollow cylinders	N_{pc}	---	4.00
73	Calculate Length ☑ **Calculate fluid temperature profile inside the borehole.**			
74				
75				
76	**Results in final iteration**			
77	short term ground thermal resistance (hourly pulse)	R_h	m-K/W	0.09
78	medium term ground thermal resistance (monthly pulse)	R_m	m-K/W	0.16
79	long term ground thermal resistance (yearly pulse)	R_y	m-K/W	0.16
80	heat pump outlet temperature	T_{outHP}	°C	36.01
81	average fluid temperature in the borehole	T_m	°C	33.00
82	length per borehole	H	m	85.00
83	total length	L_{tot}	m	1700.00
84				
85	Parameters	Rb	L (Tp)	ε
86	Units	(m-K/W)	(m (°C))	%
87	Iteration - 1	0.1	100.0 (1.35)	---
88	Iteration - 2	0.1	83.5 (1.61)	19.7
89	Iteration - 3	0.1	85.1 (1.58)	1.9
90	Iteration - 4	0.1	85.0 (1.58)	0.2
91	Final iteration	0.1	85.0 (1.58)	0.0
92	qy*Ry %	qm*Rm %	qh*Rh %	qh*Rb %
93	4.3	17.6	35.0	43.0
94				

◂ ▸ ... **L2-Classic ASHRAE Equation** | L2-Modified ASHRAE equation | L2-Alternative ASHRAE method

in brackets account for the yearly, monthly and hourly effective ground thermal resistances (i.e., R_y, R_m and R_h).

$$L = \frac{q_y\left[\left(G_{(t_3)} - G_{(t_3-t_1)}\right)/k_g\right] + q_m\left[\left(G_{(t_3-t_1)} - G_{(t_3-t_2)}\right)/k_g\right] + q_h\left[\left(G_{(t_3-t_2)}\right)/k_g\right] + q_h R_b}{\dfrac{\left(T_{in,hp} + T_{out,hp}\right)}{2} - \left(T_g + T_p\right)} \qquad (6)$$

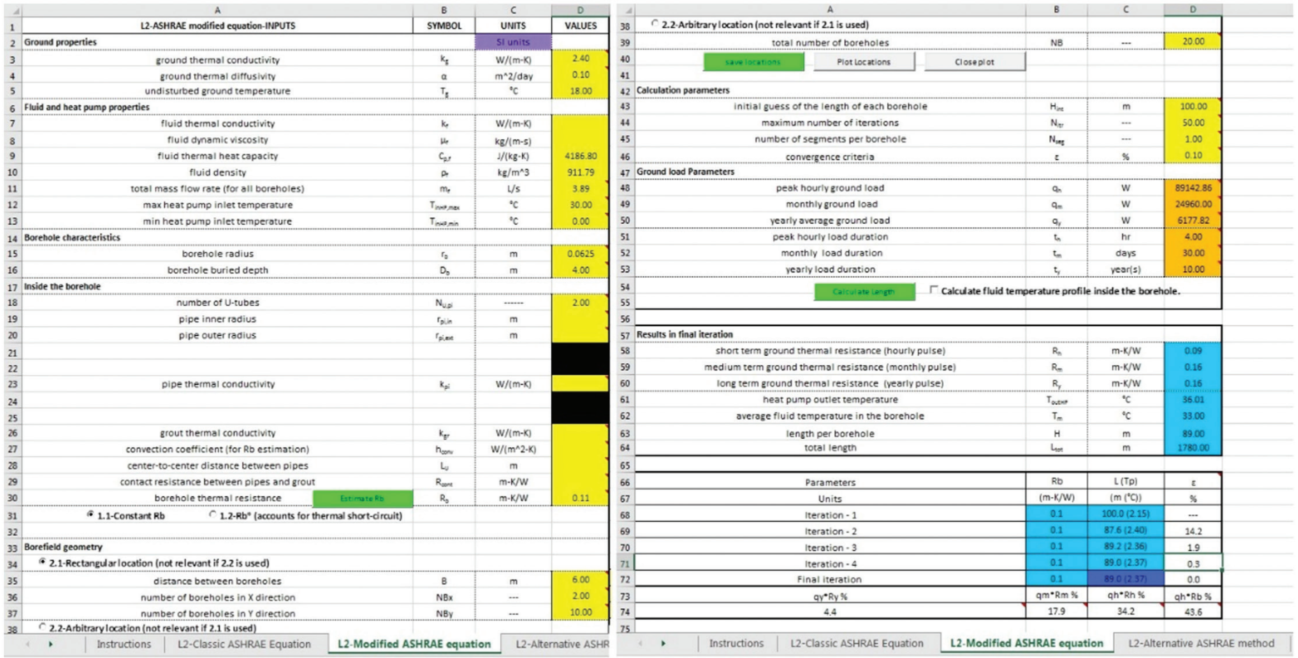
In this model, T_p is determined based on the difference between the g-function of the entire bore field and the g-function of a single borehole at a time *t* corresponding to the duration of the three ground loads.[15] This model can be used for both rectangular bore fields and bore fields with irregular borehole placement. A tab in GHXSizing is devoted to the evaluation of T_p for a given bore field geometry.

The previous example is solved again with the modified ASHRAE equation in GHXSizing, and additional inputs and results are reported in *Table 2*.

Since heat transfer varies along the borehole length, g-functions are calculated by discretizing the borehole into several segments. As explained in the companion article, calculating g-functions with a high number of segments increases the accuracy of g-functions, but adds computational time. However, as will be shown shortly, in most cases, g-functions based on one segment lead to sufficiently accurate and conservative results. In this example, the bore field is rectangular, and option 2.1 (mentioned in cell D34) is used by entering the distance between boreholes and the number of boreholes in each direction in cells D35 to D37. As can be seen in cell C72, calculations lead to a borehole length of 89.0 m (292 ft) with T_p = 2.37°C (4.27°F); both values are slightly higher than those reported in *Table 1* with the classic ASHRAE sizing equation.

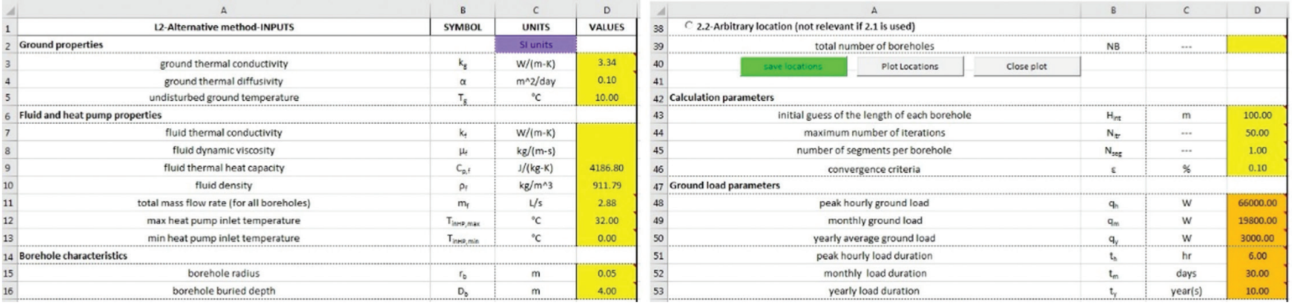
For irregular bore field geometries, option 2.2 in cell A38 should be used. In this case, the borehole locations are entered in columns G and H (not shown in *Table 2*) and recorded using the "save locations" button in cell A40 before the length is calculated.

TABLE 2 Input parameters and results obtained using the modified ASHRAE equation.



	A	B	C	D
1	**L2-ASHRAE modified equation-INPUTS**	**SYMBOL**	**UNITS**	**VALUES**
2	**Ground properties**		SI units	
3	ground thermal conductivity	k_g	W/(m-K)	2.40
4	ground thermal diffusivity	α	m^2/day	0.10
5	undisturbed ground temperature	T_g	°C	18.00
6	**Fluid and heat pump properties**			
7	fluid thermal conductivity	k_f	W/(m-K)	
8	fluid dynamic viscosity	μ_f	kg/(m-s)	
9	fluid thermal heat capacity	$C_{p,f}$	J/(kg-K)	4186.80
10	fluid density	ρ_f	kg/m^3	911.79
11	total mass flow rate (for all boreholes)	m_f	L/s	3.89
12	max heat pump inlet temperature	$T_{inHP,max}$	°C	30.00
13	min heat pump inlet temperature	$T_{inHP,min}$	°C	0.00
14	**Borehole characteristics**			
15	borehole radius	r_b	m	0.0625
16	borehole buried depth	D_b	m	4.00
17	**Inside the borehole**			
18	number of U-tubes	$N_{U,pi}$	------	2.00
19	pipe inner radius	$r_{pi,in}$	m	
20	pipe outer radius	$r_{pi,ext}$	m	
21				
22				
23	pipe thermal conductivity	k_{pi}	W/(m-K)	
24				
25				
26	grout thermal conductivity	k_{gr}	W/(m-K)	
27	convection coefficient (for Rb estimation)	h_{conv}	W/(m^2-K)	
28	center-to-center distance between pipes	L_u	m	
29	contact resistance between pipes and grout	R_{cont}	m-K/W	
30	borehole thermal resistance Estimate Rb	R_b	m-K/W	0.11
31	● 1.1-Constant Rb ○ 1.2-Rb* (accounts for thermal short-circuit)			
32				
33	**Borefield geometry**			
34	● 2.1-Rectangular location (not relevant if 2.2 is used)			
35	distance between boreholes	B	m	6.00
36	number of boreholes in X direction	NBx	---	2.00
37	number of boreholes in Y direction	NBy	---	10.00
38	○ 2.2-Arbitrary location (not relevant if 2.1 is used)			
39	total number of boreholes	NB	---	20.00
40	save locations Plot Locations Close plot			
41				
42	**Calculation parameters**			
43	initial guess of the length of each borehole	H_{int}	m	100.00
44	maximum number of iterations	N_{itr}	---	50.00
45	number of segments per borehole	N_{seg}	---	1.00
46	convergence criteria	ε	%	0.10
47	**Ground load Parameters**			
48	peak hourly ground load	q_h	W	89142.86
49	monthly ground load	q_m	W	24960.00
50	yearly average ground load	q_y	W	6177.82
51	peak hourly load duration	t_h	hr	4.00
52	monthly load duration	t_m	days	30.00
53	yearly load duration	t_y	year(s)	10.00
54	Calculate length ☐ **Calculate fluid temperature profile inside the borehole.**			
55				
56				
57	**Results in final iteration**			
58	short term ground thermal resistance (hourly pulse)	R_h	m-K/W	0.09
59	medium term ground thermal resistance (monthly pulse)	R_m	m-K/W	0.16
60	long term ground thermal resistance (yearly pulse)	R_y	m-K/W	0.16
61	heat pump outlet temperature	T_{outHP}	°C	36.01
62	average fluid temperature in the borehole	T_m	°C	33.00
63	length per borehole	H	m	89.00
64	total length	L_{tot}	m	1780.00
65				
66	Parameters	Rb	L (Tp)	ε
67	Units	(m-K/W)	(m (°C))	%
68	Iteration - 1	0.1	100.0 (2.15)	---
69	Iteration - 2	0.1	87.6 (2.40)	14.2
70	Iteration - 3	0.1	89.2 (2.36)	1.9
71	Iteration - 4	0.1	89.0 (2.37)	0.3
72	Final iteration	0.1	89.0 (2.37)	0.0
73	qy*Ry %	qm*Rm %	qh*Rh %	qh*Rb %
74	4.4	17.9	34.2	43.6
75				

TABLE 3 Input parameters and results obtained using the alternative ASHRAE method.



	A	B	C	D
1	**L2-Alternative method-INPUTS**	**SYMBOL**	**UNITS**	**VALUES**
2	**Ground properties**		SI units	
3	ground thermal conductivity	k_g	W/(m-K)	3.34
4	ground thermal diffusivity	α	m^2/day	0.10
5	undisturbed ground temperature	T_g	°C	10.00
6	**Fluid and heat pump properties**			
7	fluid thermal conductivity	k_f	W/(m-K)	
8	fluid dynamic viscosity	μ_f	kg/(m-s)	
9	fluid thermal heat capacity	$C_{p,f}$	J/(kg-K)	4186.80
10	fluid density	ρ_f	kg/m^3	911.79
11	total mass flow rate (for all boreholes)	m_f	L/s	2.88
12	max heat pump inlet temperature	$T_{inHP,max}$	°C	32.00
13	min heat pump inlet temperature	$T_{inHP,min}$	°C	0.00
14	**Borehole characteristics**			
15	borehole radius	r_b	m	0.05
16	borehole buried depth	D_b	m	4.00
38	○ 2.2-Arbitrary location (not relevant if 2.1 is used)			
39	total number of boreholes	NB	---	
40	save locations Plot Locations Close plot			
41				
42	**Calculation parameters**			
43	initial guess of the length of each borehole	H_{int}	m	100.00
44	maximum number of iterations	N_{itr}	---	50.00
45	number of segments per borehole	N_{seg}	---	1.00
46	convergence criteria	ε	%	0.10
47	**Ground load parameters**			
48	peak hourly ground load	q_h	W	66000.00
49	monthly ground load	q_m	W	19800.00
50	yearly average ground load	q_y	W	3000.00
51	peak hourly load duration	t_h	hr	6.00
52	monthly load duration	t_m	days	30.00
53	yearly load duration	t_y	year(s)	10.00

Alternative ASHRAE Method

The *2019 ASHRAE Handbook—HVAC Applications*[16] suggests a second sizing method (in addition to the classic ASHRAE equation) called the alternative ASHRAE method. In this method, the ground thermal resistances, $R_{g,y}$, $R_{g,m}$ and $R_{g,h}$ (terms in brackets in *Equation 7*) are evaluated with g-functions. Since g-functions include borehole thermal interactions caused by the annual ground thermal imbalance, the temperature penalty, T_p, is no longer needed.

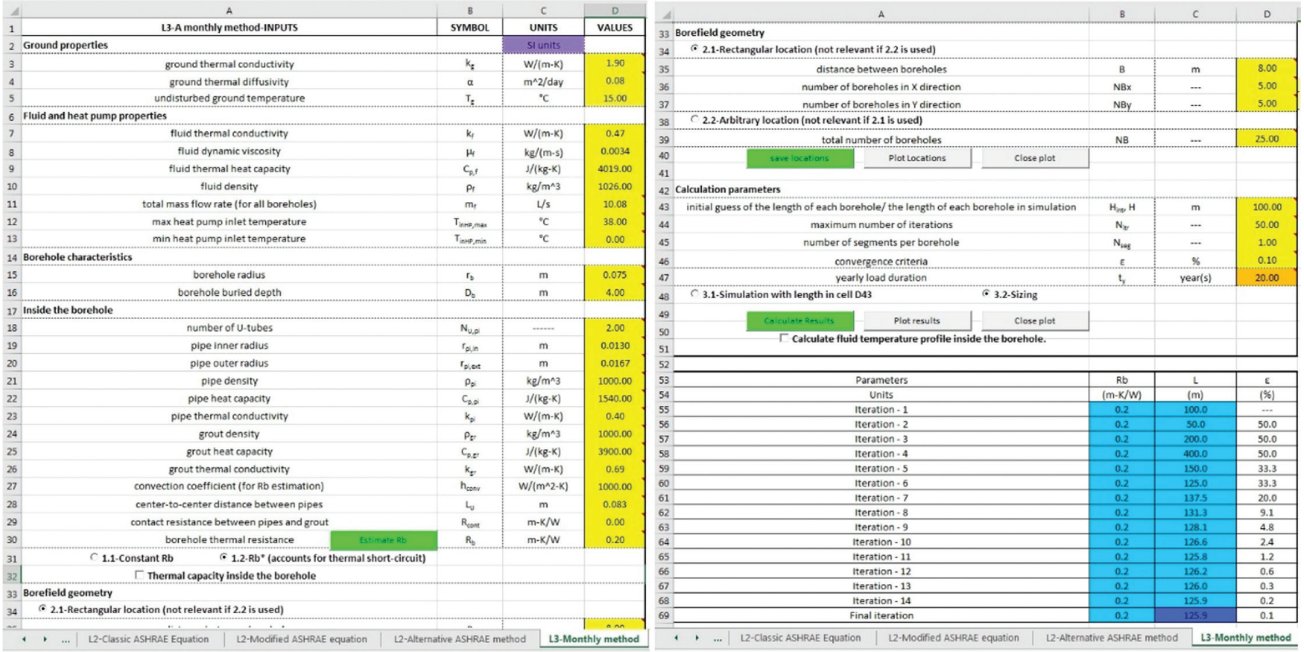
$$L = \frac{q_y\left[\left(g_{(t_3)} - g_{(t_3-t_1)}\right)/2\pi k_g\right] + q_m\left[\left(g_{(t_3-t_1)} - g_{(t_3-t_2)}\right)/2\pi k_g\right] + q_h\left[g_{(t_3-t_2)}/2\pi k_g\right] + q_h R_b}{\frac{\left(T_{in,hp} + T_{out,hp}\right)}{2} - T_g} \qquad (7)$$

Most current commercial sizing tools use libraries of precalculated g-functions with limited availability for irregular bore field layouts. However, in GHXSizing, g-functions can be evaluated for both regular and irregular configurations. In addition, evaluation of *L* with *Equation 7* involves an iterative process because g-functions depend on *L*, which is unknown a priori.

The alternative ASHRAE method is illustrated using GHXSizing to solve the second sizing problem contained in the *2019 ASHRAE Handbook—HVAC Applications*[17] for a 3 × 2 bore field. *Table 3* shows the input parameters and results for this problem.

The fluid thermal heat capacity, density, flow rate and maximum heat pump inlet temperature are set so that the maximum mean fluid temperature in the borehole, shown in cell D62, is 35°C (95°F), as given

TABLE 4 Input parameters and results obtained using the monthly sizing method for a rectangular bore field.



	A	B	C	D
1	L3-A monthly method-INPUTS	SYMBOL	UNITS	VALUES
2	Ground properties		SI units	
3	ground thermal conductivity	k_g	W/(m-K)	1.90
4	ground thermal diffusivity	α	m^2/day	0.08
5	undisturbed ground temperature	T_g	°C	15.00
6	Fluid and heat pump properties			
7	fluid thermal conductivity	k_f	W/(m-K)	0.47
8	fluid dynamic viscosity	μ_f	kg/(m-s)	0.0034
9	fluid thermal heat capacity	$C_{p,f}$	J/(kg-K)	4019.00
10	fluid density	ρ_f	kg/m^3	1026.00
11	total mass flow rate (for all boreholes)	m_f	L/s	10.08
12	max heat pump inlet temperature	$T_{inHP,max}$	°C	38.00
13	min heat pump inlet temperature	$T_{inHP,min}$	°C	0.00
14	Borehole characteristics			
15	borehole radius	r_b	m	0.075
16	borehole buried depth	D_b	m	4.00
17	Inside the borehole			
18	number of U-tubes	$N_{U,pi}$	------	2.00
19	pipe inner radius	$r_{pi,in}$	m	0.0130
20	pipe outer radius	$r_{pi,ext}$	m	0.0167
21	pipe density	ρ_{pi}	kg/m^3	1000.00
22	pipe heat capacity	$C_{p,pi}$	J/(kg-K)	1540.00
23	pipe thermal conductivity	k_{pi}	W/(m-K)	0.40
24	grout density	ρ_{gr}	kg/m^3	1000.00
25	grout heat capacity	$C_{p,gr}$	J/(kg-K)	3900.00
26	grout thermal conductivity	k_{gr}	W/(m-K)	0.69
27	convection coefficient (for Rb estimation)	h_{conv}	W/(m^2-K)	1000.00
28	center-to-center distance between pipes	L_U	m	0.083
29	contact resistance between pipes and grout	R_{cont}	m-K/W	0.00
30	borehole thermal resistance Estimate Rb	R_b	m-K/W	0.20
31	○ 1.1-Constant Rb ● 1.2-Rb* (accounts for thermal short-circuit)			
32	☐ Thermal capacity inside the borehole			
33	Borefield geometry			
34	● 2.1-Rectangular location (not relevant if 2.2 is used)			

◂ ▸ ... L2-Classic ASHRAE Equation | L2-Modified ASHRAE equation | L2-Alternative ASHRAE method | L3-Monthly method

	A	B	C	D
33	Borefield geometry			
34	● 2.1-Rectangular location (not relevant if 2.2 is used)			
35	distance between boreholes	B	m	8.00
36	number of boreholes in X direction	NBx	---	5.00
37	number of boreholes in Y direction	NBy	---	5.00
38	○ 2.2-Arbitrary location (not relevant if 2.1 is used)			
39	total number of boreholes	NB	---	25.00
40	save locations Plot Locations Close plot			
41				
42	Calculation parameters			
43	initial guess of the length of each borehole/ the length of each borehole in simulation	H_{ini}, H	m	100.00
44	maximum number of iterations	N_{itr}	---	50.00
45	number of segments per borehole	N_{seg}	---	1.00
46	convergence criteria	ε	%	0.10
47	yearly load duration	t_y	year(s)	20.00
48	○ 3.1-Simulation with length in cell D43 ● 3.2-Sizing			
49	Calculate Results Plot results Close plot			
50	☐ Calculate fluid temperature profile inside the borehole.			
51				
52				
53	Parameters	Rb	L	ε
54	Units	(m-K/W)	(m)	(%)
55	Iteration - 1	0.2	100.0	---
56	Iteration - 2	0.2	50.0	50.0
57	Iteration - 3	0.2	200.0	50.0
58	Iteration - 4	0.2	400.0	50.0
59	Iteration - 5	0.2	150.0	33.3
60	Iteration - 6	0.2	125.0	33.3
61	Iteration - 7	0.2	137.5	20.0
62	Iteration - 8	0.2	131.3	9.1
63	Iteration - 9	0.2	128.1	4.8
64	Iteration - 10	0.2	126.6	2.4
65	Iteration - 11	0.2	125.8	1.2
66	Iteration - 12	0.2	126.2	0.6
67	Iteration - 13	0.2	126.0	0.3
68	Iteration - 14	0.2	125.9	0.2
69	Final iteration	0.2	125.9	0.1

◂ ▸ ... L2-Classic ASHRAE Equation | L2-Modified ASHRAE equation | L2-Alternative ASHRAE method | L3-Monthly method

in the *Handbook* example. The g-functions are evaluated with one segment. GHXSizing results in cell C70 show that the borehole length is 99.8 m (327.4 ft), which is the value reported in the *ASHRAE Handbook*. Calculations for these input parameters with the classic and modified ASHRAE equations (not shown in this article) give borehole lengths of 100.9 m (331.0 ft) and 101.4 m (332.7 ft), respectively. Thus, all three methods give essentially the same results for this case. However, as shown by Ahmadfard and Bernier,[2] the classic ASHRAE equation starts to diverge from the modified ASHRAE equation and the alternative ASHRAE method for large annual ground thermal imbalance.

GHXSizing offers the possibility to account for the influence of borehole thermal capacity (fluid, pipe and grout) on sizing. This additional feature, not described in the *Handbook*, can be activated in cell A32. When the feature is used, borehole thermal capacity effects are calculated using short-term g-functions.[18] Considering borehole thermal capacity reduces the required borehole length by a few percent, especially if the peak duration is short.

In these first three sizing models, it is assumed that the length is required at the end of the design period. However, in some cases, the required borehole length may be required in the first months of operation, which may lead to an undersized bore field if these models are used. One way to circumvent this issue is to use either L3 or L4 sizing methods. These two methods will now be described.

Hourly and Monthly Sizing Methods

The governing equations for the monthly (L3) and hourly (L4) sizing methods are also g-function–based. They can be derived from *Equation 4* and are presented in the instructions tab in GHXSizing. Much like *Equation 7*, these governing equations are solved iteratively in GHXSizing by updating borehole lengths until either the maximum or minimum heat pump inlet temperatures are reached. During these iterations, g-functions are evaluated dynamically as the solution progresses. For the hourly sizing method, a multiple load aggregation algorithm[19] is used to limit calculation time.

The monthly and hourly sizing methods available in GHXSizing are illustrated here with two geometries involving 25 boreholes and a relatively high annual ground thermal imbalance. The geometries are illustrated in *Figure 4*. The first geometry is a square bore field, while the second geometry has a pentagonal shape. In both cases, borehole spacing is 8.0 m (26.2 ft). The monthly and hourly ground loads of this problem can be found in GHXSizing as well as in Ahmadfard and Bernier.[2]

Table 4 shows the input parameters and results when the bore field is rectangular and the monthly method (L3) is used. The monthly average ground loads are determined by averaging all hourly loads during that month. The monthly peak ground loads are the maximum hourly loads in a given month, and they are assumed here to last six hours. As shown in cell C69 in *Table 4*, the calculated borehole length is 125.9 m (413 ft) for the rectangular configuration.

Figure 5a shows the evolution of the solution and indicates that convergence is obtained after 15 iterations. *Figure 5b* illustrates the evolutions of the maximum and minimum inlet fluid temperatures for the final borehole length (125.9 m [413 ft]). It can be seen that the heat pump inlet fluid temperature reaches its maximum design value of 38°C (100.4°F) during July of the twentieth year. As for the minimum inlet fluid temperature, a value of 9°C (48.2°F), which is much higher than the minimum design fluid temperature of 0°C (32°F), is reached during the first year of operation.

It can be shown that the pentagonal shape leads to a shorter length of 106.2 m (348 ft) per borehole. This is because there is less borehole thermal interaction as the average distance between boreholes is greater than for the rectangular shape. The length difference observed between the rectangular and pentagonal shapes is in line with the conclusions of a recent study by Spitler, et al.,[20] who have shown that when compared to regularly spaced configurations, irregular bore field geometries can significantly reduce the total drilling length of a bore field while yielding the same performance.

Table 5 examines variations on this sizing problem for the rectangular geometry. Increasing the number of segments to 12 for the determination of g-functions decreases the length to 122.6 m (402 ft) for the rectangular configuration, a reduction of 2.6%. In addition, if short-term effects (i.e., borehole thermal capacity) are considered, the borehole length is reduced to 118.1 m (388 ft), a reduction of 6.2% from the base case. The hourly method leads to a borehole length of 123.7 m (406 ft).

It is also interesting to compare these results to those obtained with L2 methods (assuming a peak duration of 6 hours). As shown in *Table 5*, the modified ASHRAE equation and the alternative ASHRAE equation give borehole lengths of 128.7 m (422 ft) and 125.5 m (412 ft), respectively, close to the results obtained with the higher-level methods. Finally, it is to be noted that this problem was solved using two commercially available software tools, which

FIGURE 4 Geometries used in the third sizing problem (8 m corresponds to 26.2 ft).

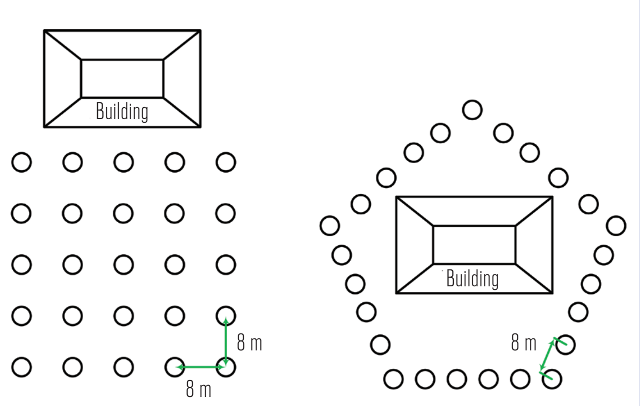
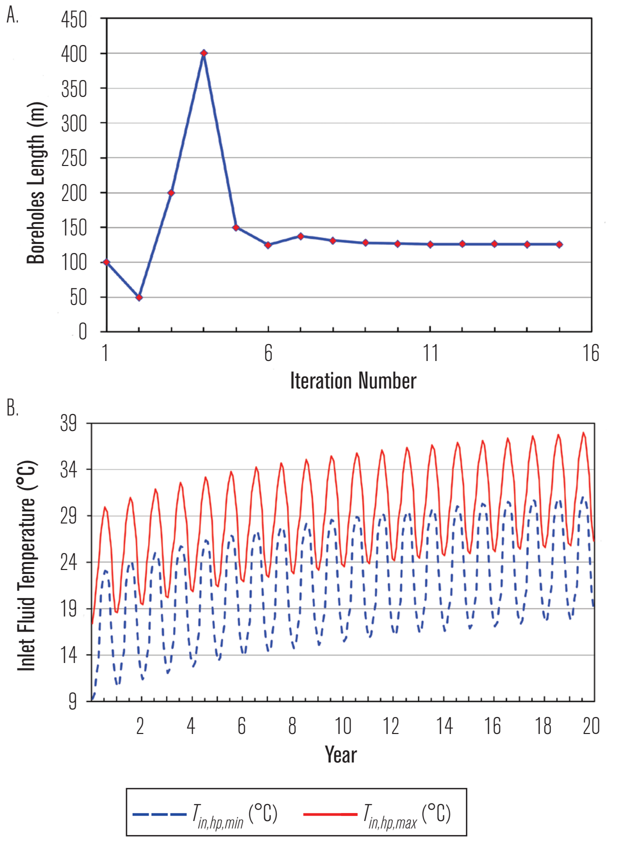


FIGURE 5 (A) Evolution of the borehole length during the solution process and (B) fluid temperature evolution for the L3 (monthly) sizing model.



calculated lengths of 123.0 m (404 ft) and 118.5 m (389 ft), respectively.[2]

TABLE 5 Detailed results for the third sizing problem using GHXSizing.

METHOD	SHORT-TERM EFFECTS	NUMBER OF BOREHOLE SEGMENTS	BOREHOLE LENGTH (m)	PERCENTAGE CHANGE
Monthly Sizing Method L3	×	1	125.9	–
Monthly Sizing Method L3	×	12	122.6	−2.6
Monthly Sizing Method L3	✓	12	118.1	−6.2
Hourly Sizing Method L4	×	1	123.7	−1.7
Modified ASHRAE Equation L2	×	1	128.7	+2.2
Alternative ASHRAE Equation L2	×	1	125.5	−0.3

Concluding Remarks

This article proposes a multilevel approach for sizing VGHEs. Most of the methods presented here rely on ground thermal response factors, also known as g-functions, which are the subject of a follow-up article. Low-level approaches use only three ground load pulses and can be used at an early design stage when limited information about the ground loads is available. Higher-level approaches use monthly or hourly loads to size the bore field. The sizing equations and the example contained in the *2019 ASHRAE Handbook—HVAC Applications* are revisited with GHXSizing, a spreadsheet tool that accompanies this article and can size rectangular and nonrectangular bore fields. It is shown that the discretization (number of segments) used in the determination of the g-functions influences the required borehole length.

In one example, the calculated borehole length is reduced by 2.6% when more precise g-functions are determined with 12 segments instead of one. It is shown that short-term or borehole thermal capacity effects, which are not yet treated in the *2019 ASHRAE Handbook—HVAC Applications* sizing methods, tend to reduce the required borehole length by a few percent. Finally, it is observed that low- and high-level methods give similar results when they use the same load profile (*Table 5*).

Supplementary Material

A spreadsheet tool called GHXSizing accompanies this article. It can be found at on ASHRAE's website at https://tinyurl.com/mbmdpbv4 and on the author's website (https://tinyurl.com/GeoSizing).

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